The Currency-Maturity Composition of Sovereign Debt

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How should governments borrow?

Two important aspects of a government's debt portfolio are:

1. **Currency:** What currency is the sovereign borrowing & repaying in?
   Barro (1979), Calvo (1988), Eichengreen and Hausmann (1999), Ottonello and Perez (2019),Du et al. (2020), Rebelo et al. (2022), Engel and Park (2022)
2. **Maturity**: How long before the sovereign must repay the bond?
   Calvo and Guidotti (1990), Missale and Blanchard (1991) , Arellano and Ramanarayanan (2012), Broner et al. (2013), Aguiar et al. (2019), Bornstein (2020), Bhattarai et al. (2022)

Debt trade-offs

Two mechanisms determine the optimal portfolio:

1. Hedging vs Incentive benefits
2. Price

	Hedging	**Incentive**	**Price**
Local currency (LC)	✓		
Foreign currency (FC)		✓	✓
Long term (LT)	✓		
Short term (ST)		✓	✓

This paper

First in the literature to examine both aspects of sovereign debt **jointly**.

Research question: What is the optimal currency-maturity composition of sovereign debt?

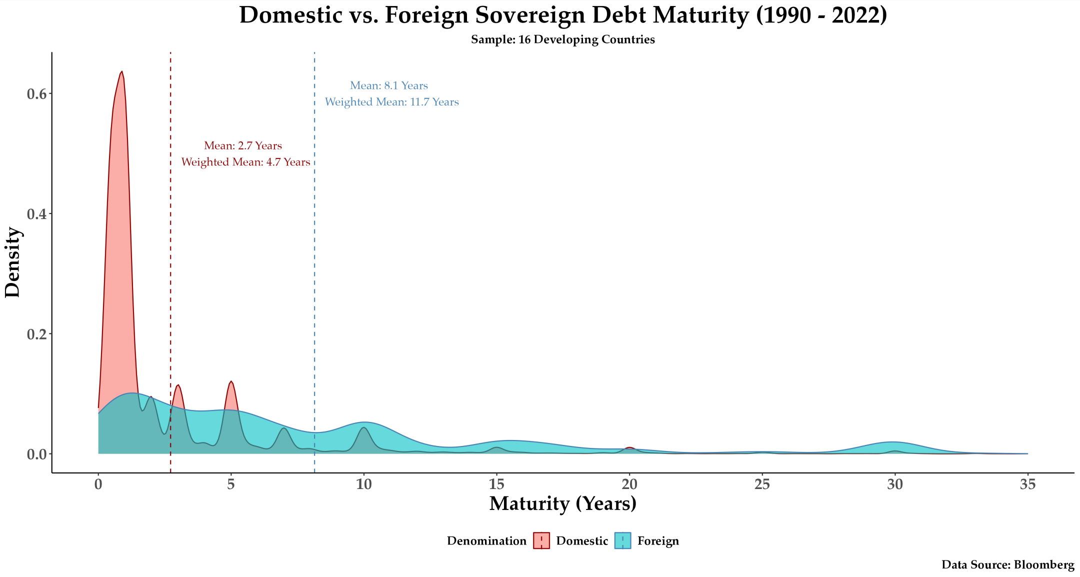
Data

List of countries

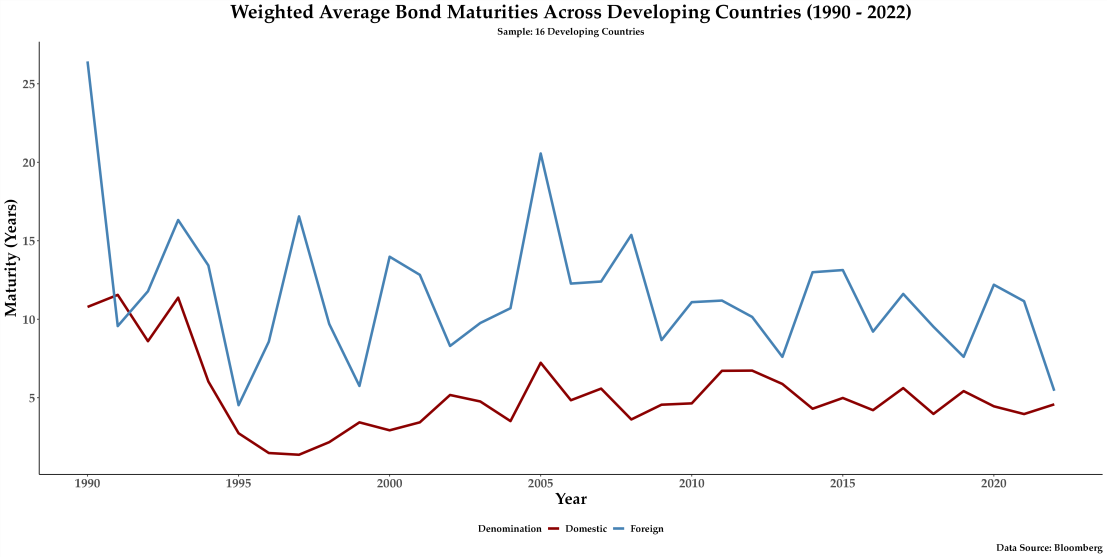
- Bloomberg bond *issuance* data
- January 1st 1990 - December 31st 2022
- 15 developing economies
- $n = 21,532$ bond issuances, after filtering out
    - Bonds with maturities less than 3 months
    - Bonds issued by municipal/sub-national entities
    - Bonds issued by central banks

Stylized facts - maturity composition

Country	All	Domestic	Foreign
Argentina	6.4 years	3.5 years	7.9 years
Egypt	1.6 years	1.4 years	4.4 years
Jamaica	6.8 years	6.7 years	7.9 years
Jordan	3.4 years	3.2 years	9.4 years
Mexico	3.9 years	3.0 years	11.7 years
Philippines	3.1 years	2.5 years	12.0 years
Poland	3.3 years	2.2 years	11.2 years
Romania	2.3 years	1.6 years	8.4 years
Russia	6.2 years	5.7 years	11.4 years
Ukraine	3.3 years	3.2 years	4.1 years
Venezuela	3.5 years	1.7 years	15.9 years
Average	3.8 years	**3.0 years**	**8.3 years**
Median	3.4 years	**2.9 years**	**7.9 years**



Domestic vs. Foreign Sovereign Debt Maturity (1990 - 2022)
Sample: 16 Developing Countries
Mean: 8.1 Years
Weighted Mean: 11.7 Years
Mean: 2.7 Years
Weighted Mean: 4.7 Years
Density
0.6
0.4
0.2
0.0
0
5
10
15
20
25
30
35
Maturity (Years)
Denomination
Domestic
Foreign
Data Source: Bloomberg


Weighted Average Bond Maturities Across Developing Countries (1990 - 2022)
Sample: 16 Developing Countries
Maturity (Years)
25
20
15
10
5
1990
1995
2000
2005
2010
2015
2020
Year
Denomination
Domestic
Foreign
Data Source: Bloomberg

Model environment

- Small open economy
- Infinite horizon
- Stochastic endowment economy
- Endogenous strategic default
- No international inflation
- Law of one price holds
- Borrowing only from international investors

Social planner (SP)'s problem

- Representative household preferences:

$$\max \sum_{t=0}^{\infty} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left\{ u(C_t) - l(\Pi_t) \right\} \right], \quad 0 < \beta < 1$$

  subject to:

  1. SP's budget constraint
  2. $\ln(y_{t+1}) = \rho \ln(y_t) + \varepsilon_{t+1}, \varepsilon_{t+1} \sim$ I.I.D $\mathcal{N}(0, \sigma_\varepsilon^2)$
  3. Borrowing limit (that never binds in equilibrium)

- C_t: Period t consumption
- P_t: Period t price level
- $\Pi_t \equiv \frac{P_t}{P_{t-1}}$: Period t inflation
- $u : \mathbb{R}_+ \to \mathbb{R}$: differentiable, increasing, concave function
- $l : \mathbb{R}_+ \to \mathbb{R}$: differentiable, convex function

SP's problem

In each period, after observing y_t, the SP can either:

1. Strategically default on its debt obligations
    - but suffer output loss: $y^{\text{default}} = h(y)$ where $0 \leq h(y) \leq y$, &
    - financial autarky, with a θ probability of being able to borrow again
2. Repay by choosing the optimal combination of inflation (Π_t) and 4 bonds
    - Short-term debt in local currency: $b_{s,t+1}$
    - Short-term debt in foreign currency: $b^*_{s,t+1}$
    - Long-term debt in local currency: $b_{l,t+1}$
    - Long-term debt in foreign currency: $b^*_{l,t+1}$

Bonds

- $B' > 0$ indicates saving, $B' < 0$ indicates borrowing
- Perpetuity contract structure from Hatchondo and Martinez (2009)
- Coupon payments that decay geometrically at rate δ
  - Issue 1 bond in foreign (local) currency in period t
  - Receive $q_t^*(q_t)$ units of foreign (local) currency in period t
  - Repay δ^{j-1} unit in of foreign (local) currency in period $t+j$
- Short-term bonds nested as $\delta = 0$
- Laws of motion:

$$
\begin{aligned}
b_{l,t+1} &= \delta b_{l,t} + i_{l,t} \\
b^*_{l,t+1} &= \delta b^*_{l,t} + i^*_{l,t} \\
b_{s,t+1} &= i_{s,t} \\
b^*_{s,t+1} &= i^*_{s,t}
\end{aligned}
$$

SP's budget constraint

$$P_t C_t - \overbrace{e_t(b^*_{s,t} + b^*_{l,t}) - b_{s,t} - b_{l,t}}^{\text{Period } t \text{ debt obligations}} = P_t Y_t - \overbrace{e_t q^*_{l,t}(b^*_{l,t+1} - \delta b^*_{l,t}) + q_{l,t}(b_{l,t+1} - \delta b_{l,t})}^{\text{Newly issued LT debt}} \tag{1}$$

$$- \underbrace{e_t q^*_{s,t} b^*_{s,t+1} - q_{s,t} b_{s,t+1}}_{\text{Newly issued ST debt}}$$

Recursive formulation of the de-trended problem

- Let $\mathbf{S} = \{Y, b_s, b_s^*, b_l, b_l^*\}$ denote the aggregate state.
- Let $v^d(y)$ denote the value function associated with defaulting.
- Let $v^c(\mathbf{S})$ denote the value function associated with repaying.
- Let $v^o(\mathbf{S})$ denote the SP's value function when deciding whether to default or not default.

Value functions

- The value of defaulting is:

$$v^d(y) = u(h(y)) + \beta \int \left[\theta v^o(y', 0, 0, 0, 0) + (1-\theta) v^d(y')\right] f(y, y') dy'$$

- The value of repaying is:

$$v^c(\mathbf{S}) = \max_{\{b_s', b_s^{*\prime}, b_l', b_l^{*\prime}, \Pi, C\}} \left\{ u(C) - l(\Pi) + \beta \int v^o(\mathbf{S}') f(y, y') dy' \right\}$$

subject to:

- 4 bond pricing functions, &

$$C - \frac{b_s + b_l}{\Pi} - b_s^* - b_l^* = y - q_s(y, \mathbf{B}') b_s' - q_s^*(y, \mathbf{B}') b_s^{*\prime}$$
$$-q_l(y, \mathbf{B}') \left(b_l' - \frac{\delta b_l}{\Pi} \right) - q_l^*(y, \mathbf{B}') \left(b_l^{*\prime} - \delta b_l^* \right)$$

Value functions

- Together, the three value functions are linked by:

$$v^o(\mathbf{S}) = \max_{c,d} \left\{ v^c(\mathbf{S}), v^d(y) \right\}$$

Let the:

1. **Repayment set** be the set of income levels for which repaying is optimal when the overall debt level is given by (b_s, b_l, b_s^*, b_l^*):

$$R(b_s, b_l, b_s^*, b_l^*) = \left\{ y : v^c(\mathbf{S}) \geq v^d(y) \right\}$$

2. **Default set** by the set of income levels for which default is optimal when the overall debt level is given by

$$D(b_s, b_l, b_s^*, b_l^*) = \left\{ y : v^c(\mathbf{S}) < v^d(y) \right\}$$

Bond pricing

- All bonds are priced by perfectly competitive, risk-neutral, foreign investors.
- Can access a riskless FC security that pays gross international interest rate R^*.
- Receive 0 if the SP decides to default.
- Let $\mathbf{B}' = \{b_s', b_s^{*\prime}, b_l', b_l^{*\prime}\}$ denote the vector of bond issuances.

$$q_s^*(y, \mathbf{B}') = \frac{1}{R^*} \int_{R(\mathbf{B}')} f(y, y') dy'$$

$$q_l^*(y, \mathbf{B}') = \frac{1}{R^*} \int_{R(\mathbf{B}')} [1 + \delta q_l^*(y', \mathbf{B}'')] f(y, y') dy'$$

$$q_s(y, \mathbf{B}') = \frac{1}{R^*} \int_{R(\mathbf{B}')} \left[\frac{1}{\Pi'(\mathbf{S}')} \right] f(y, y') dy'$$

$$q_l(y, \mathbf{B}') = \frac{1}{R^*} \int_{R(\mathbf{B}')} \left[\frac{1}{\Pi'(\mathbf{S}')} [1 + \delta q_l'(y', \mathbf{B}'')] \right] f(y, y') dy'$$

Markov perfect equilibrium (MPE)

A MPE consists of:

- the value functions $\left\{v^o(\mathbf{S}), v^c(\mathbf{S}), v^d(\mathbf{S})\right\}$,
- a set of policy functions $\{C(\mathbf{S}), \Pi(\mathbf{S}), b_s'(\mathbf{S}), b_l'(\mathbf{S}), b_s^{*\prime}(\mathbf{S}), b_l^{*\prime}(\mathbf{S})\}$, &
- bond prices $\{q_s(y, \mathbf{B}'), q_l(y, \mathbf{B}'), q_s^*(y, \mathbf{B}'), q_l^*(y, \mathbf{B}')\}$

such that:

1. Taking the bond pricing functions as given, the default decision rule and policies $\{C(\mathbf{S}), \Pi(\mathbf{S}), b_s'(\mathbf{S}), b_l'(\mathbf{S}), b_s^{*\prime}(\mathbf{S}), b_l^{*\prime}(\mathbf{S})\}$ solves the SP's optimization problem, &
2. Prices $\{q_s(y, \mathbf{B}'), q_l(y, \mathbf{B}'), q_s^*(y, \mathbf{B}'), q_l^*(y, \mathbf{B}')\}$ reflect default probabilities and inflation risk.

Numerical strategy

0. Set upper and lower bounds on $\mathbf{S}$. Given these bounds, construct a μ-level Smolyak grid and associated polynomials following Smolyak's method.
1. Provide an initial guess of $v^c(\mathbf{S}), v^d(\mathbf{S})$ (and implicitly, $v^o(\mathbf{S})$).
2. Find the cut-off rule $\bar{y}(\mathbf{B})$ such that $v^c(\mathbf{S}) < v^d(\mathbf{S})$.
3. Pre-compute default probabilities:

$$P\left(\underbrace{y^\rho \exp(\varepsilon)}_{y'} < \bar{y}(\mathbf{B}')\right) = F\left(\ln\left(\frac{\bar{y}(\mathbf{B}')}{y^\rho}\right)\right)$$

   where F is the CDF of a normal distribution.
4. Pre-compute $q_s^*(y, \mathbf{B}')$:

$$q_s^*(y, \mathbf{B}') = \frac{1}{R^*}\left[1 - F\left(\ln\left(\frac{\bar{y}(\mathbf{B}')}{y^\rho}\right)\right)\right]$$

Numerical strategy

Conditioning on not defaulting,

1. 5 FOCs (4 for the 4 bonds, 1 for Π),
2. 4 envelope conditions (for 4 bonds), &
3. 1 resource constraint.

Combining the 4 FOCs and 4 ECs results in a system of 6 residual equations:

1. 4 consumption-Euler equations for bonds,
2. 1 intra-temporal optimality condition for Π, &
3. 1 resource constraint

for the 6 controls $\left\{C(\hat{\mathbf{S}}), \Pi(\hat{\mathbf{S}}), b_s'(\hat{\mathbf{S}}), b_l'(\hat{\mathbf{S}}), b_s^{*\prime}(\hat{\mathbf{S}}), b_l^{*\prime}(\hat{\mathbf{S}})\right\}$ to solve each period.

Numerical strategy

5. Guess an inflation policy function $\hat{\Pi}(\mathbf{S})$
6. Solve for $v^c(\mathbf{S})$ and update $v^d(y)$ according to
7. If $v^c(\mathbf{S})$ and $v^d(y)$ converged, end iteration. Otherwise, repeat Step 1 - Step 10.

Appendix: List of developing countries

List of countries

- **Africa:** Egypt
- **Asia:** Indonesia, Malaysia, Philippines
- **Americas:** Argentina, Jamaica, Mexico, Venezuela
- **Europe:** Hungary, Poland, Romania, Russia, Turkey, Ukraine
- **Middle East:** Jordan

Back to data

References

Aguiar, Mark, Manuel Amador, Hugo Hopenhayn, and Iván Werning, "Take the short route: Equilibrium default and debt maturity," *Econometrica*, 2019, *87* (2), 423–462.

Arellano, Cristina and Ananth Ramanarayanan, "Default and the maturity structure in sovereign bonds," *Journal of Political Economy*, 2012, *120* (2), 187–232.

Barro, Robert J, "On the determination of the public debt," *Journal of Political Economy*, 1979, *87* (5, Part 1), 940–971.

Bhattarai, Saroj, Gauti B Eggertsson, and Bulat Gafarov, "Time consistency and duration of government debt: A model of quantitative easing," *Review of Economic Studies*, 2022, pp. 1–41.

Bornstein, Gideon, "A continuous-time model of sovereign debt," *Journal of Economic Dynamics and Control*, 2020, *118*, 103963.

Broner, Fernando A, Guido Lorenzoni, and Sergio L Schmukler, "Why do emerging economies borrow short term?," *Journal of the European Economic Association*, 2013, *11* (suppl_1), 67–100.

Calvo, Guillermo A, "Servicing the public debt: The role of expectations," *American Economic Review*, 1988, pp. 647–661.

__ **and Pablo E Guidotti**, "Credibility and nominal debt: exploring the role of maturity in managing inflation," *Staff Papers*, 1990, *37* (3), 612–635.

Du, Wenxin, Carolin E Pflueger, and Jesse Schreger, "Sovereign debt portfolios, bond risks, and the credibility of monetary policy," *Journal of Finance*, 2020, *75* (6), 3097–3138.

Eichengreen, Barry and Ricardo Hausmann, "Exchange rates and financial fragility," 1999.

Engel, Charles and JungJae Park, "Debauchery and original sin: The currency composition of sovereign debt," *Journal of the European Economic Association*, 2022, *20* (3), 1095–1144.

Hatchondo, Juan Carlos and Leonardo Martinez, "Long-duration bonds and sovereign defaults," *Journal of international Economics*, 2009, *79* (1), 117–125.

Missale, Alessandro and Olivier J Blanchard, "The debt burden and debt maturity," *American Economic Review*, 1991, *84* (1), 309–319.

Ottonello, Pablo and Diego J Perez, "The currency composition of sovereign debt," *American Economic Journal: Macroeconomics*, 2019, *11* (3), 174–208.

Rebelo, Sergio, Neng Wang, and Jinqiang Yang, "Rare disasters, financial development, and sovereign debt," *The Journal of Finance*, 2022, *77* (5), 2719–2764.